Pre-AP Geometry Date_____ 4.6 Notes

Isosceles, Equilateral, and Right Triangles (pp 236-237)

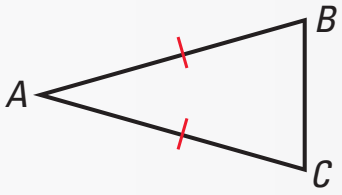
- **I can solve problems and perform proofs with isosceles, right and equilateral triangles.**

THEOREMS

THEOREM 4.6 *Base Angles Theorem*

If two sides of a triangle are congruent, then the angles opposite them are congruent.

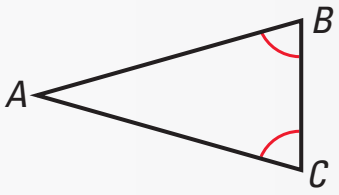
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



THEOREM 4.7 *Converse of the Base Angles Theorem*

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.



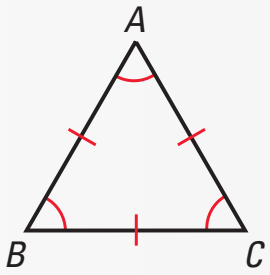
COROLLARIES

COROLLARY TO THEOREM 4.6

If a triangle is equilateral, then it is equiangular.

COROLLARY TO THEOREM 4.7

If a triangle is equiangular, then it is equilateral.

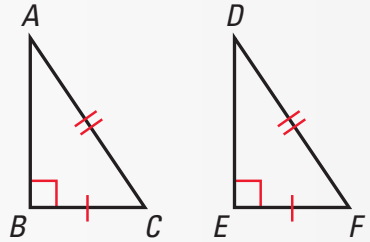


THEOREM

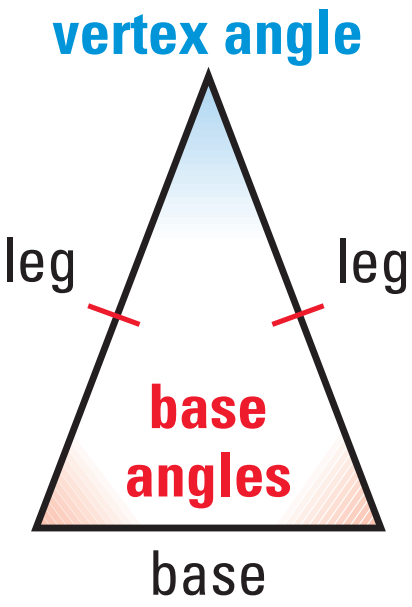
THEOREM 4.8 *Hypotenuse-Leg (HL) Congruence Theorem*

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.



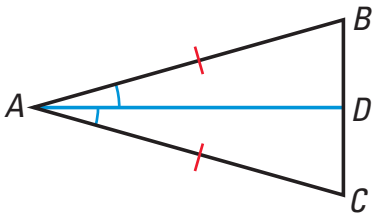
Pre-AP Geometry Date____ 4.6 Notes
Isosceles, Equilateral, and Right Triangles (pp 236-237)



EXAMPLE 1 ***Proof of the Base Angles Theorem***

Proof

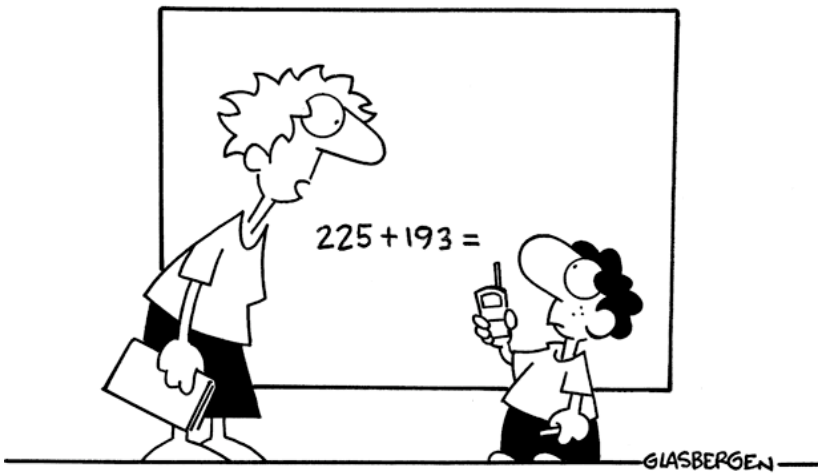
Use the diagram of $\triangle ABC$ to prove the Base Angles Theorem.



GIVEN ▶ $\triangle ABC$, $\overline{AB} \cong \overline{AC}$

PROVE ▶ $\angle B \cong \angle C$

Paragraph Proof Draw the bisector of $\angle CAB$. By construction, $\angle CAD \cong \angle BAD$. You are given that $\overline{AB} \cong \overline{AC}$. Also, $\overline{DA} \cong \overline{DA}$ by the Reflexive Property of Congruence. Use the SAS Congruence Postulate to conclude that $\triangle ADB \cong \triangle ADC$. Because corresponding parts of congruent triangles are congruent, it follows that $\angle B \cong \angle C$.



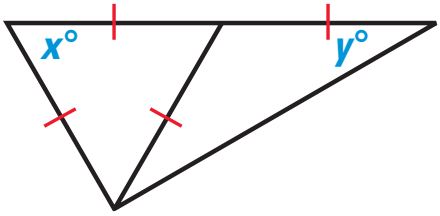
"You have to solve this problem by yourself. You can't call tech support."

EXAMPLE 2 *Using Equilateral and Isosceles Triangles*

a. Find the value of x.

b. Find the value of y.

Using Algebra



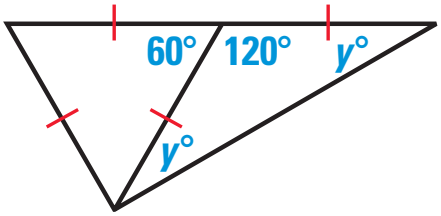
SOLUTION

a. Notice that x represents the measure of an angle of an equilateral triangle. From the corollary above, this triangle is also equiangular.

$3x° = 180°$ **Apply the Triangle Sum Theorem.**

$x = 60$ **Solve for *x*.**

b. Notice that y represents the measure of a base angle of an isosceles triangle. From the Base Angles Theorem, the other base angle has the same measure. The vertex angle forms a linear pair with a 60° angle, so its measure is 120°.

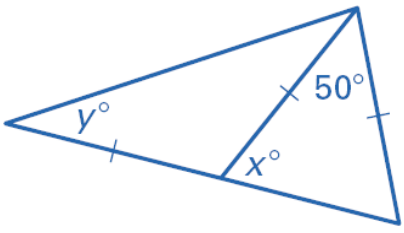


$120° + 2y° = 180°$ **Apply the Triangle Sum Theorem.**

$y = 30$ **Solve for *y*.**

2. Find the value of x.

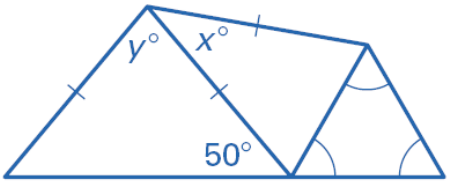
3. Find the value of y.



Pre-AP Geometry Date_____ 4.6 Notes
Isosceles, Equilateral, and Right Triangles (pp 236-237)

4. Find the value of x.

5. Find the value of y.

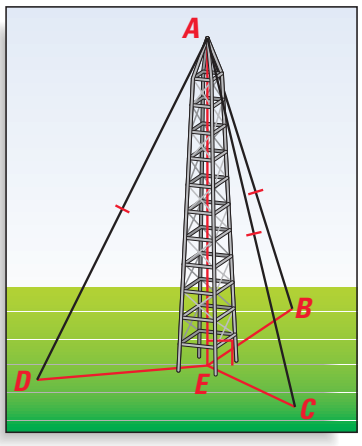


Ways to Prove triangles are congruent.

- Side-Side-Side (SSS) Congruence Postulate (p. 212)
- Side-Angle-Side (SAS) Congruence Postulate (p. 213)
- Angle-Side-Angle (ASA) Congruence Postulate (p. 220)
- Angle-Angle-Side (AAS) Congruence Theorem (p. 220)

EXAMPLE 3 ***Proving Right Triangles Congruent***

The television antenna is perpendicular to the plane containing the points B, C, D, and E. Each of the stays running from the top of the antenna to B, C, and D uses the same length of cable. Prove that $\triangle AEB$, $\triangle AEC$, and $\triangle AED$ are congruent.



GIVEN ▶ $\overline{AE} \perp \overline{EB}$, $\overline{AE} \perp \overline{EC}$, $\overline{AE} \perp \overline{ED}$, $\overline{AB} \cong \overline{AC} \cong \overline{AD}$

PROVE ▶ $\triangle AEB \cong \triangle AEC \cong \triangle AED$

SOLUTION

Paragraph Proof You are given that $\overline{AE} \perp \overline{EB}$ and $\overline{AE} \perp \overline{EC}$, which implies that $\angle AEB$ and $\angle AEC$ are right angles. By definition, $\triangle AEB$ and $\triangle AEC$ are right triangles. You are given that the hypotenuses of these two triangles, $\overline{AB}$ and $\overline{AC}$, are congruent. Also, $\overline{AE}$ is a leg for both triangles, and $\overline{AE} \cong \overline{AE}$ by the Reflexive Property of Congruence. Thus, by the Hypotenuse-Leg Congruence Theorem, $\triangle AEB \cong \triangle AEC$.

▶ Similar reasoning can be used to prove that $\triangle AEC \cong \triangle AED$. So, by the Transitive Property of Congruent Triangles, $\triangle AEB \cong \triangle AEC \cong \triangle AED$.

STUDENT HELP

Study Tip
Before you use the HL Congruence Theorem in a proof, you need to prove that the triangles are right triangles.

Pre-AP Geometry Date_____ 4.6 Notes
Isosceles, Equilateral, and Right Triangles (pp 236-237)

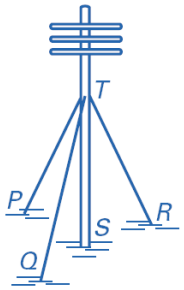
6.

Given: $\overline{TS} \perp \overline{PS}$

$\overline{TS} \perp \overline{QS}$

$\overline{TS} \perp \overline{RS}$

$\overline{TP} \cong \overline{TQ} \cong \overline{TR}$



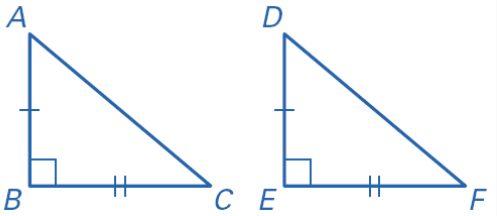
Prove: $\triangle TPS \cong \triangle TQS \cong \triangle TRS$

Guided Practice.

7. Given: $\overline{AB} \cong \overline{DE}$

$\overline{BC} \cong \overline{EF}$

$\measuredangle ABC$ & $\measuredangle DEF$ are right angles.

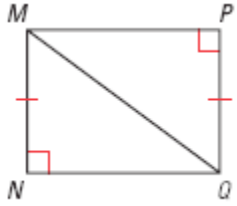


Prove: $\triangle ABC \cong \triangle DEF$

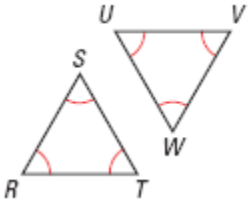
Pre-AP Geometry Date_____ 4.6 Notes
Isosceles, Equilateral, and Right Triangles (pp 236-237)

Determine whether you are given enough information to prove that the triangles are congruent. Explain your answer.

8.



9.



10.